

# Centroid moment tensor inversions using 3-D Green's tensor and adjoint methods for earthquakes included in full waveform tomography of the South Island region, New Zealand

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## 1. Introduction

In this poster, we determine centroid moment tensor (CMT) solutions by two different approaches: iterative inversion based on the adjoint-wavefield method and direct inversion from the strain Green's tensor and observed data. Since the heterogeneity of a 3-D velocity model can introduce error in source-parameter inversions based on 1-D models, it is necessary for source inversion along with structure inversion in regions with complicated crustal structure, such as the upper South Island, New Zealand. Synthetic waveforms are simulated through solving the 3-D visco-elastic wave equations (Graves, 1996) in the software emod3d and the construction of the strain Green's tensors and seismograms from reciprocal principles (Graves, 2001). We perform the ground-motion simulation for a domain of 352x352x160 km utilizing an inverted velocity model ( $m_{10}$ ) refined to 2 km grid (Figure 1) from full waveform tomography study (Nguyen, 2020).

Synthetic tests are carried out to verify each of the inversion methods. For broadband data inversions, the adjoint CMT inversions are in good agreement with the direct CMT inversion using 3-D Green's functions. The revised CMT solutions then were included for further tomography inversion to improve the 3-D velocity model.

## 2. Methodology and synthetic study

### 2.1 CMT inversion using strain Green's tensors (SGT)

A methodology was implemented to perform source inversion from seismic waveforms using strain Green's tensors (SGT) calculated for a 3-D velocity structure. Using reciprocity, these SGT can be recombined to represent the seismograms at each receiver. To obtain the SGT for a given receiver, we need three finite-difference simulations to generate 6 strain tensors from all three orthogonal-unit-impulsive point forces acting at the receiver. By saving the 18 independent elements of the SGT from 3 simulations, we can reconstruct the synthetic waveform with the same accuracy as the forward simulations.

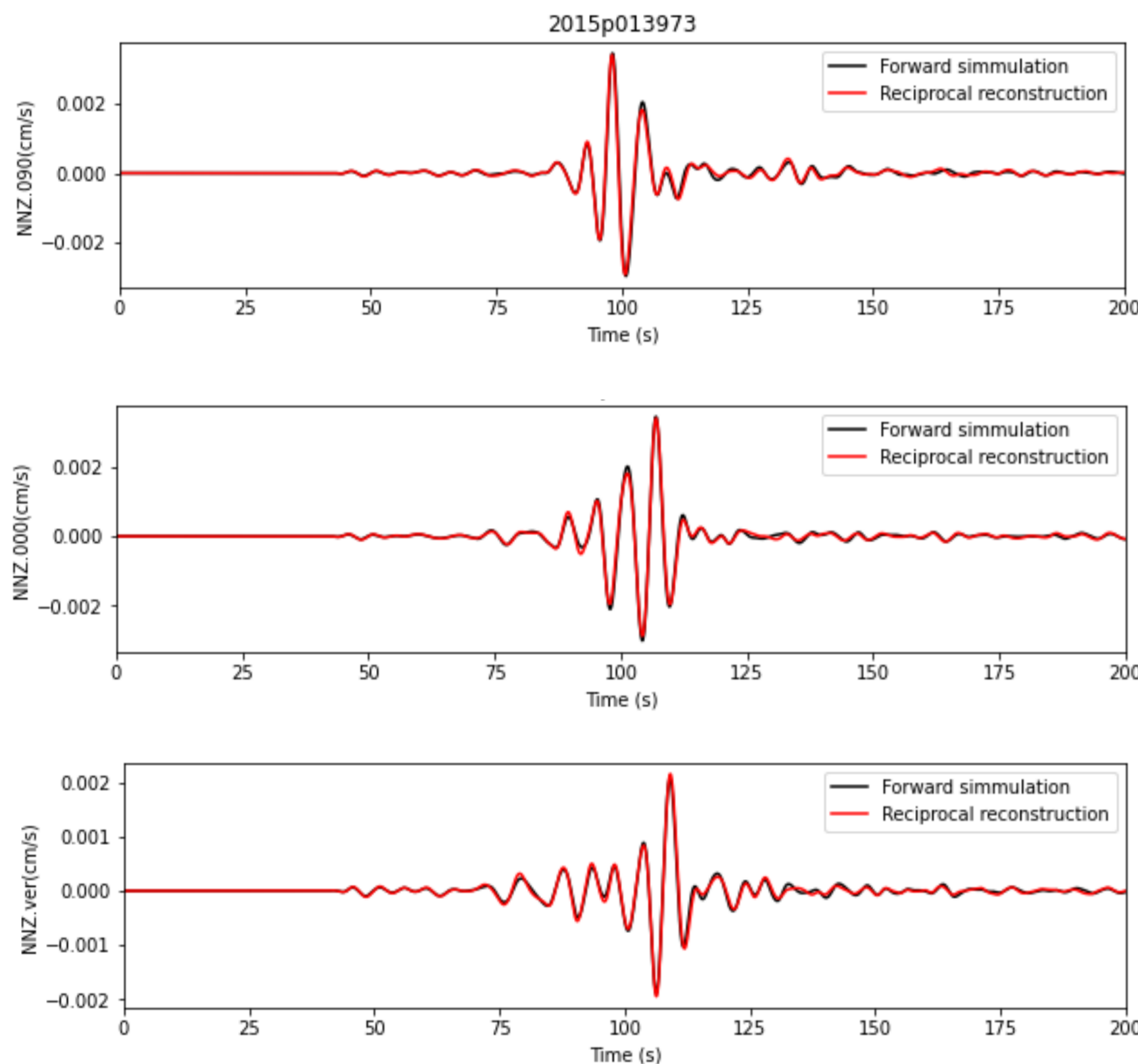


Figure 2: Synthetic waveforms from forward modelling (black) and reciprocity (red).

To demonstrate the CMT inversion using SGT, we first generated the synthetic seismograms from forward simulation (Graves, 1996) using a source solution given by GeoNet (Ristau, 2008). The recorded time was 200 s; and time step was 0.08 s and data were recorded at 10 broadband stations in the area. For SGT construction, 18 SGT components were stored for each of the 10 stations. The SGT were then combined with 6 components of centroid moment tensors from the source to form the reconstructed waveforms. Figure 2 shows the good agreement between the forward simulated and reciprocally reconstructed velocity waveforms filtered from 0.025-0.2 Hz at station NNZ for all 3 directions.

We then perform a direct inversion of the given source from the SGTs and synthetic waveforms at 10 stations using the Cut and Paste method (Zhu, 2016) with the given source locations. Figure 3 shows the good recovering of the CMT solution compared to the GeoNet solution in strike/dip/rake. Since the SGT can be stored at any cell in the domain, a grid search near the given source location can improve the source location as well.

### 2.2 CMT inversion using adjoint method

Since numerical simulations of 3-D Green's functions are computationally expensive and some complicate sources only can be solved by an iterative approach, we also implement the adjoint centroid-moment tensor inversion method (Kim, 2011). Similar to adjoint inversion for structure (Nguyen, 2020), we define a misfit function  $\chi$  (Multi-taper misfit, Tape, 2009) and calculate the Fréchet derivatives of the misfit function for source parameters including moment tensor  $M$  and centroid location  $x$  (Kim, 2011). We then non-dimensionalize the source parameters and gradients, and update the source model using a conjugate-gradient method starting from an initial solution.

To verify the performance of the method, we consider a synthetic test to invert a known source solution from the synthetic waveforms generated with a true CMT solution. For the same domain, we have a source at the center of the domain with the hypocenter depth = 8 km, Mw = 5.5 and strike/dip/rake = 90/45/90. The synthetic waveforms are recorded at 40 stations uniformly distributed on two circles with radius of 30 and 60 km from the source epicenter (Figure 4). For the initial source, we perturbed the true solution by +/- 15 degrees for strike/dip/rake correspondingly.

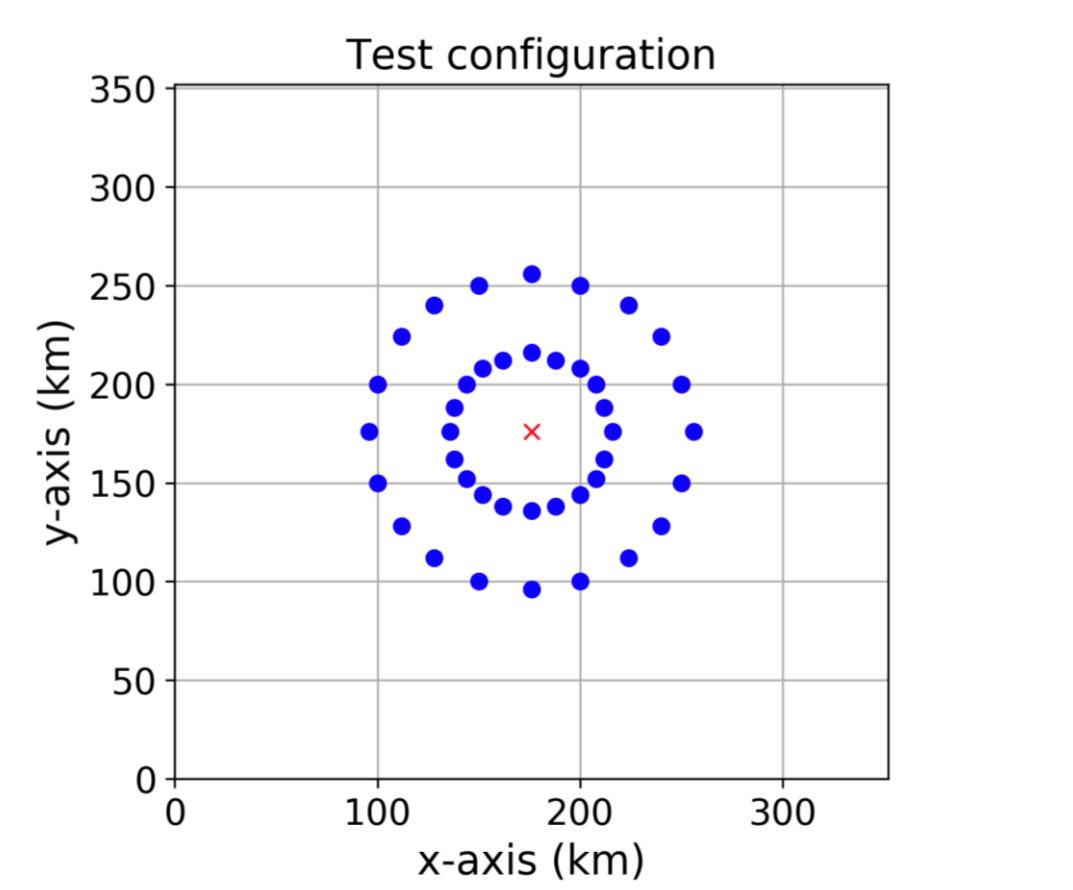


Figure 4: Source (red) and stations (blue) setup

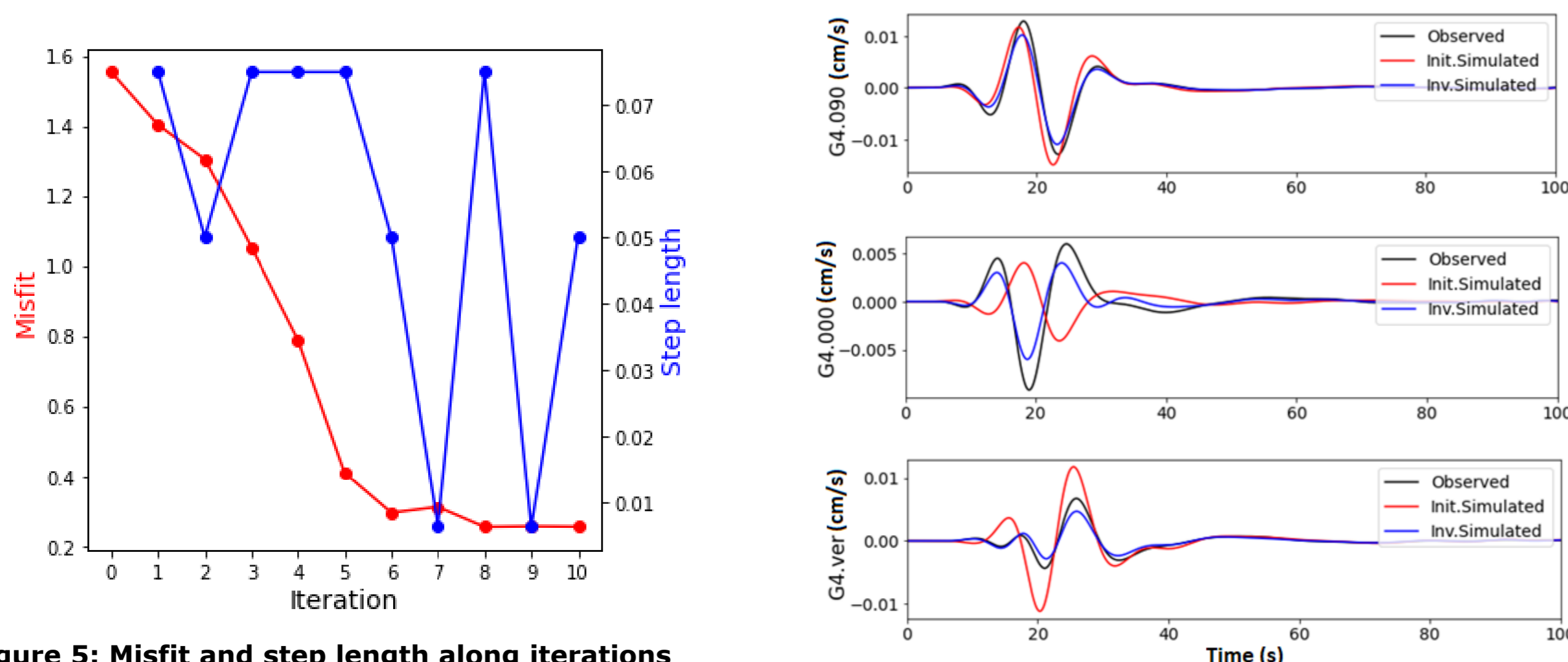


Figure 5: Misfit and step length along iterations

Figure 6: Waveform comparison

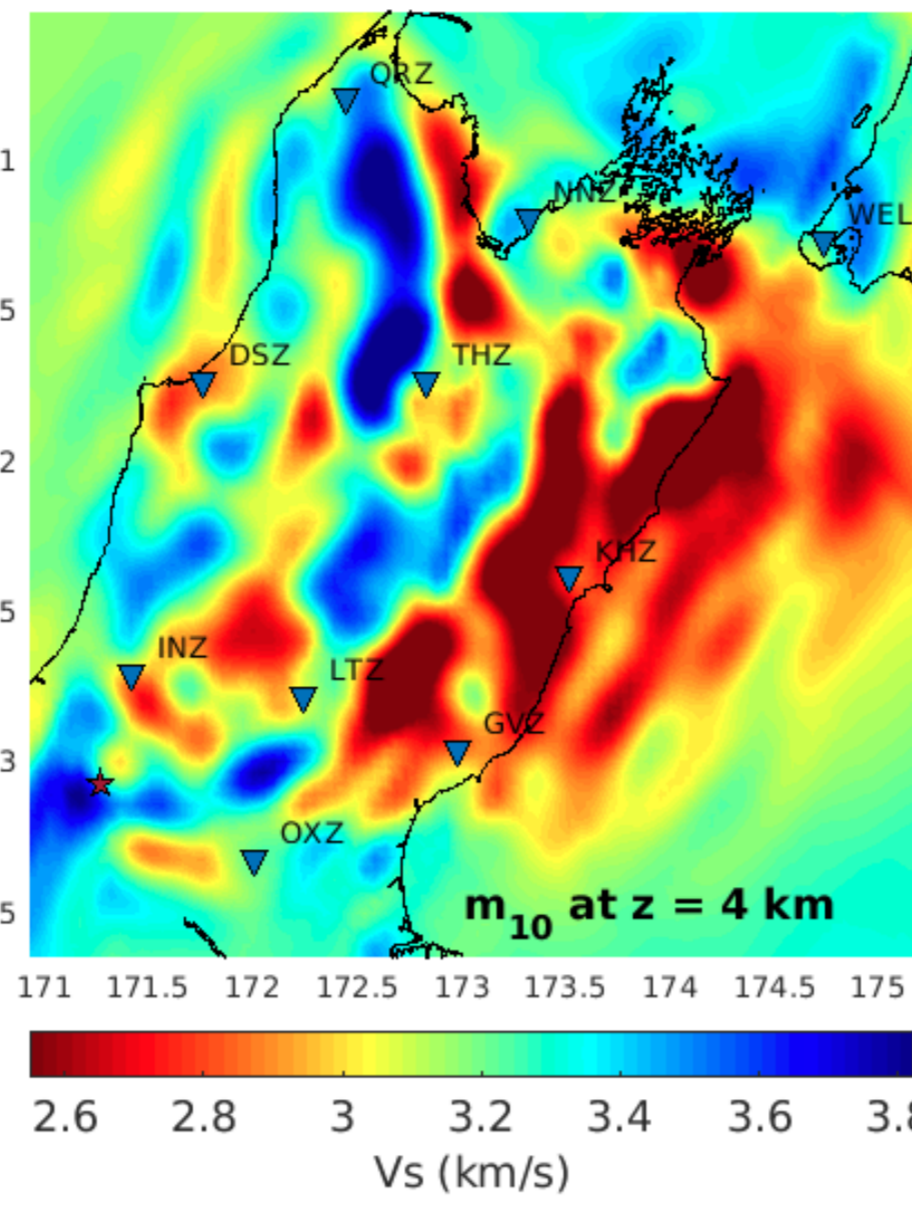


Figure 1: Vs profile of  $m_{10}$  model at 4 km depth with source (star) and broadband stations (triangles);

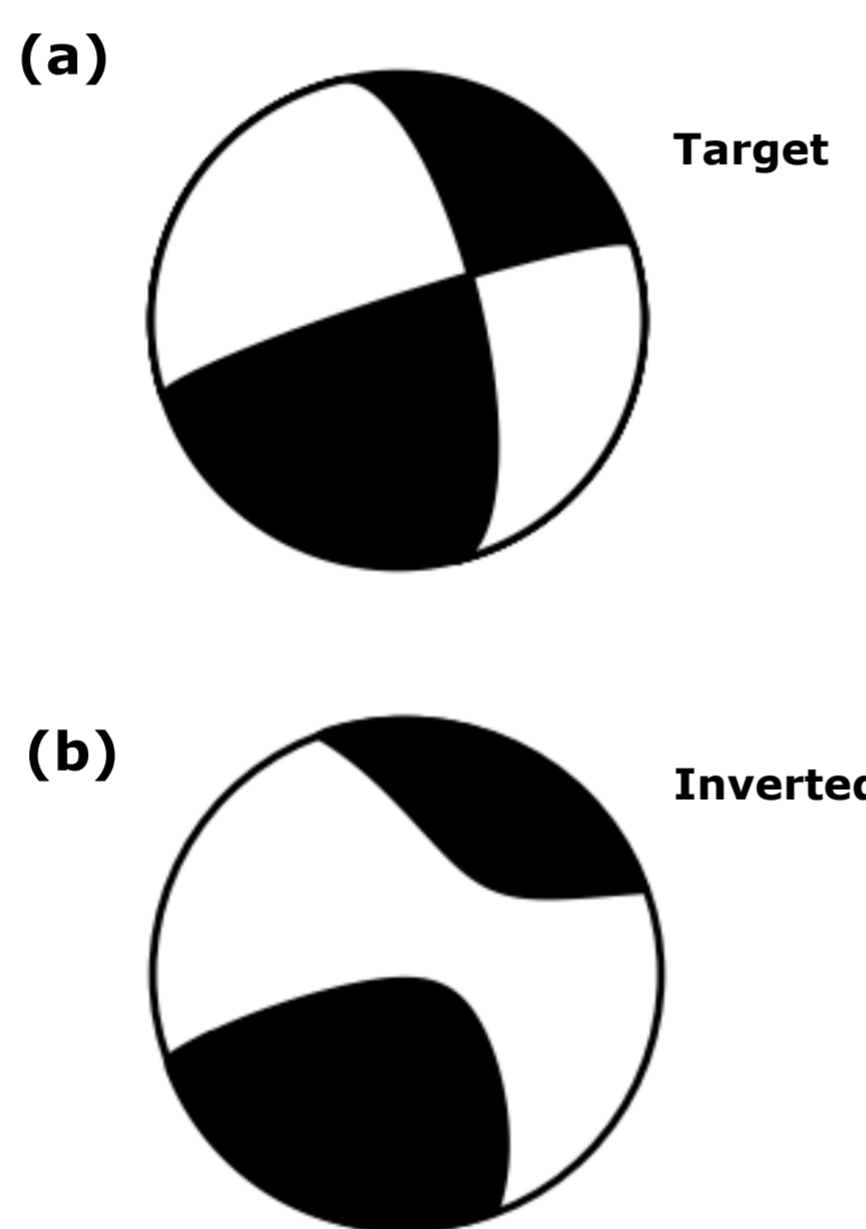
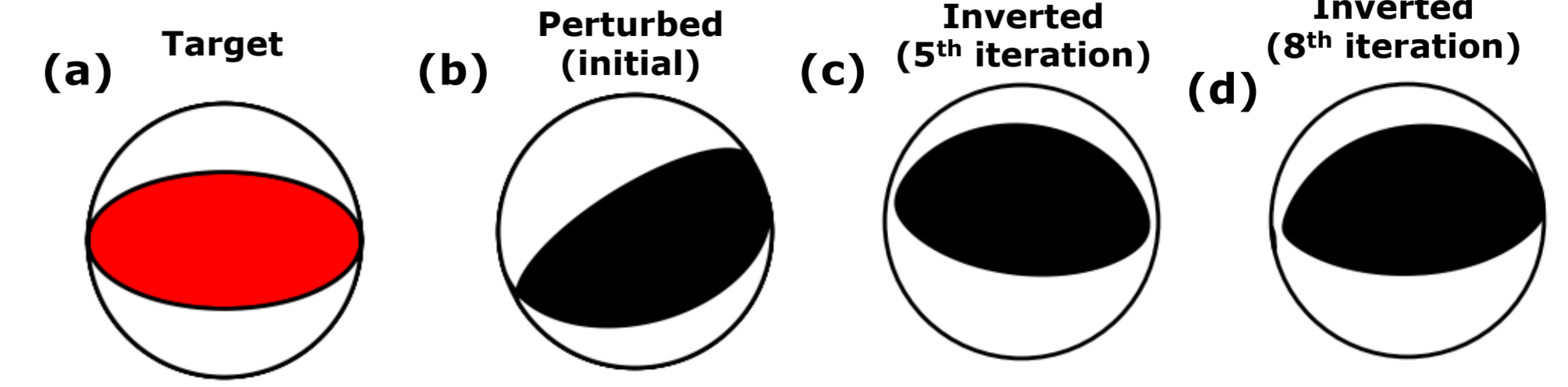


Figure 3: (a) GeoNet double-couple solution: strike/dip/rake = 347/60/10; (b) Inverted full CMT solution: strike/dip/rake = 342/68/6.

Figure 7: (a) True solution: strike/dip/rake=90/45/90; (b) Perturbed solution strike/dip/rake=75/30/105. (c) Inverted full CMT solution after 5 iteration: strike/dip/rake = 90/31/96 (d) Solution after 8 iterations: strike/dip/rake = 89/31/89



The adjoint inversion of CMT solution for the synthetic test quickly converges after 8 iterations with the misfit reducing by 88% (Figure 5) and the synthetic waveform improvement compared to the observed data (Figure 6). The inverted solutions are shown together with the true and initial solutions (Figure 7). Since the same velocity model was used to generate the observed and synthetic data, the inversion has largely improved their amplitude difference (Figure 6). Because of the computational efficiency, the adjoint method can be implemented for multi-event source revision in full waveform tomography inversion.

## 3. Application to broadband station data

### 3.1 Broadband data inversion for CMT solution using SGTs

From SGT constructed for 10 stations mentioned in Section 2.1, we can use the broadband data recorded for a specific event to revise the CMT solution. We need to perform the seismogram segmentation and carefully choose station combination to use in the inversion. This selection can be done using *pyflex* software for pair of observed and synthetic waveforms generated with a known CMT solution provided by GeoNet. We implement the strategy for source inversion to the event 3505099 previously used in the full waveform inversion of model  $m_{10}$ . Using *Pyflex*, we have selected 4 stations: QRZ, WEL, KHZ and INZ with all 3 components included. The data were filtered from 0.025-0.2 Hz. The inverted CMT solution was close to the GeoNet solution (Figure 8) and improved the waveform fitting between the observed and simulated data (Figure 9).

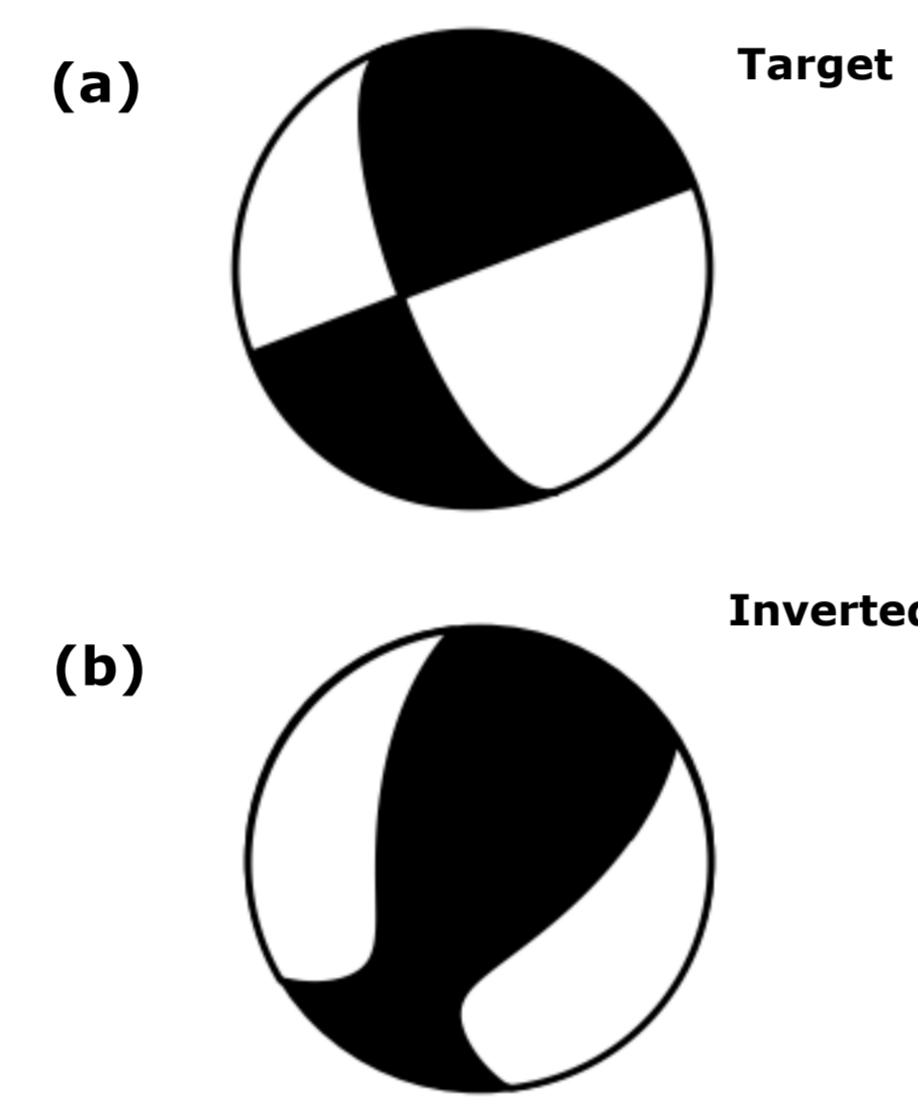


Figure 8: Event 3505099 (a) GeoNet double-couple solution: strike/dip/rake = 159/61/0; (b) Inverted full CMT solution: strike/dip/rake = 174/42/37.

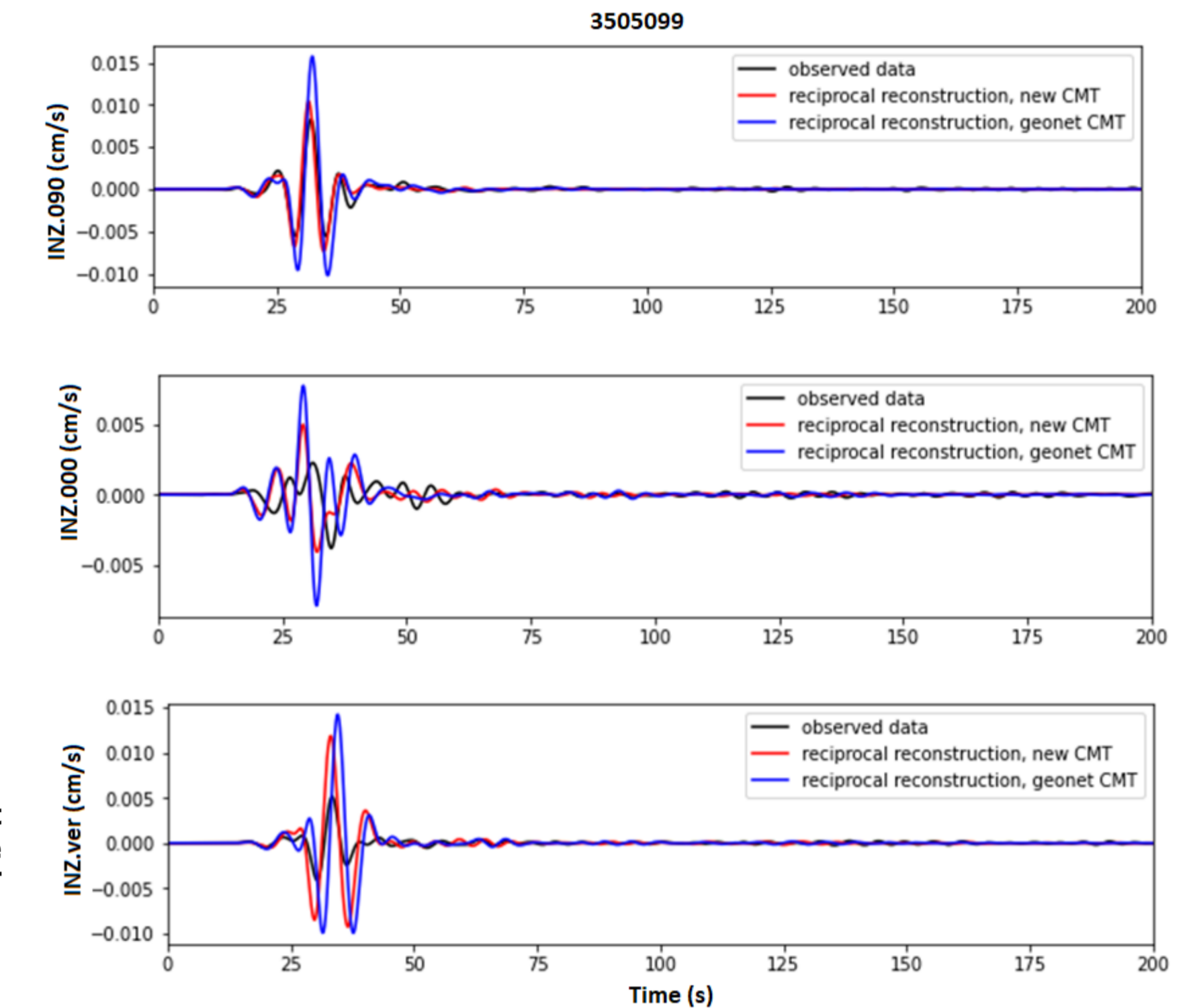


Figure 9: Waveform comparison for 3 components of the velocity seismograms according to event 3505099, station INZ.

### 3.2 CMT revised solutions by the adjoint method for improving tomographic inversion

The broadband data recorded by 10 broadband stations from 13 earthquakes with Mw 4.8-5.2 and hypocenter varying from 4 to 20 km depth according to GeoNet catalogue were utilized in the tomographic inversion of the model  $m_{10}$ . Since the tomographic inversion converged after 10 iterations, further improvement may require additional data by including more events or revising the CMT solutions of the sources corresponding to the current velocity model. By applying the adjoint CMT inversion method, we are able to revise the CMT solutions including the hypocenter depth for all 13 events. Figure 10 shows the misfit reduction along 5 iterations for 13 events using conjugate-gradient method. For each event, the revised CMT solution and hypocenter depth were selected at the iteration with the smallest misfit.

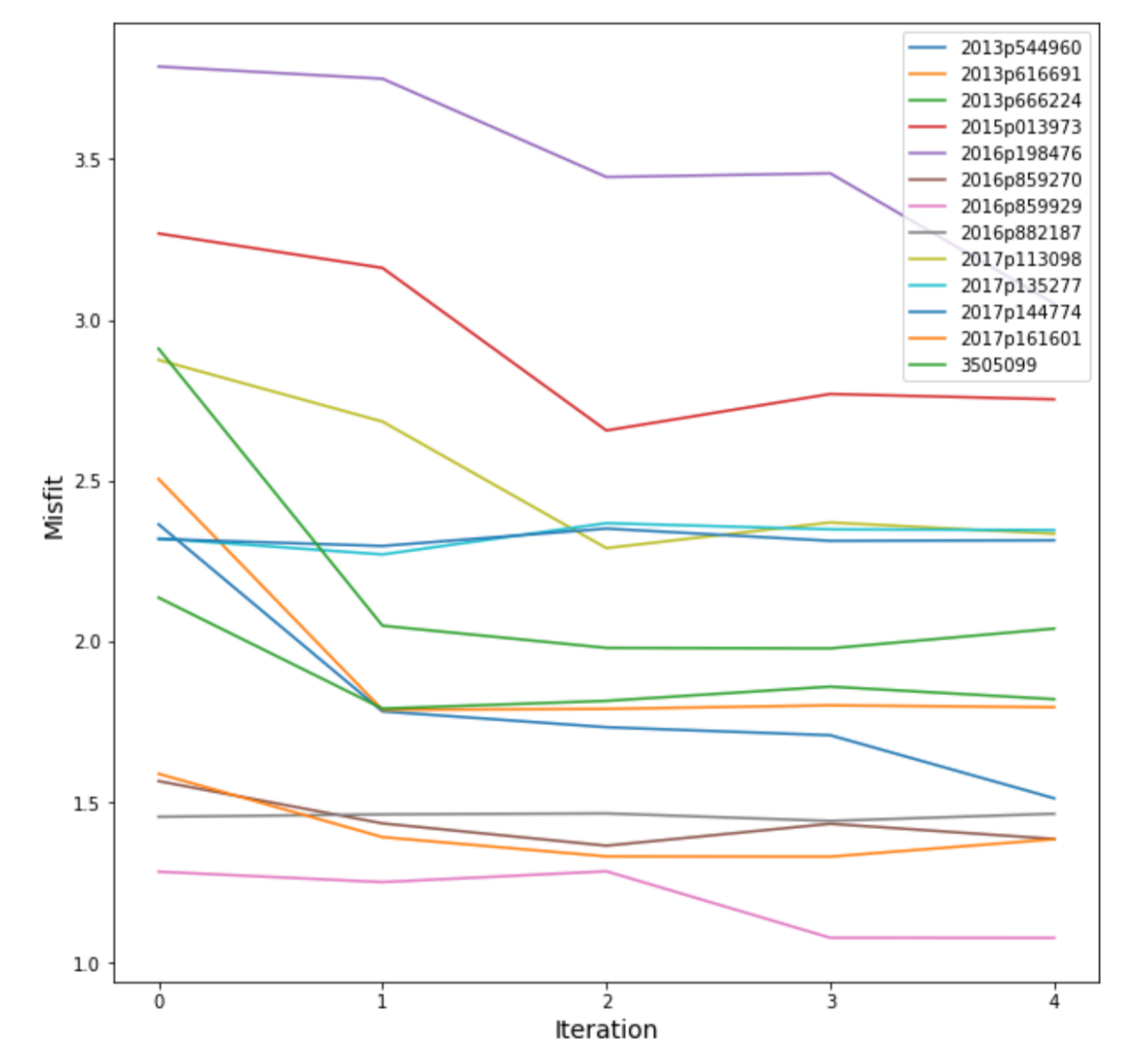


Figure 10: Misfit reduction after 5 iterations of CMT inversion using adjoint method for 13 events

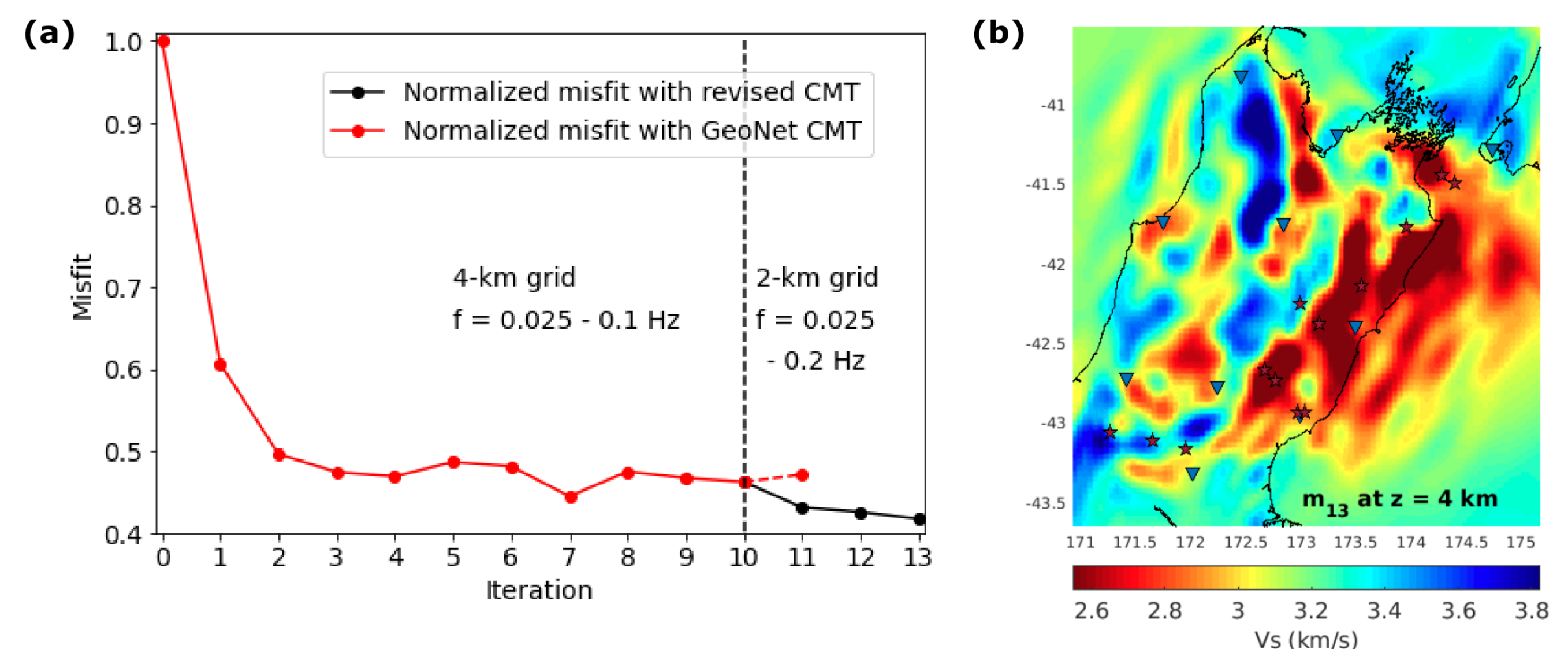


Figure 11: (a) misfit reduction along tomographic inversion using revised CMT solution starting with model  $m_{10}$ ; (b) Vs profile of inverted model  $m_{13}$  at 4 km depth

With the revised CMT solution and hypocenter depth of the sources, we can further perform tomographic inversion. The initial misfit between the observed and synthetic data was reduced significantly according to the revised CMT solution (Figure 11a). The velocity model can be improved for 3 iterations (Figure 11b) with a slight misfit reduction along iterations 12 and 13.

## 4. Conclusion

We revised the GeoNet CMT solutions for events included in the tomographic inversion using 3-D Green's functions and adjoint methods. The adjoint method was preferable because of the computational efficiency and it's ability to be incorporated to the tomographic inversion workflow. The inversion method using SGT however can derive a solution without an initial solution required in the adjoint method. Both methods can be combined for revision of a large number of earthquakes for tomographic study as well as ground motion prediction.

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### Acknowledgements

The numerical simulations were performed on Nesi's supercomputer. The work was funded by University of Canterbury and QuakeCoRE. The collaboration with University of Southern California was included in the inversion method using SGTs. We thank Donna Eberhard-Phillips for providing the background model NZVM 2010 included in the tomographic inversion and GeoNet for providing the earthquake catalogue and broadband station data used in this study.